Likelihood for Alfvénic bifurcation in experiments

Vinícius Duarte[1,2]

In collaboration with
Nikolai Gorelenkov[1], Herb Berk[3], Eric Fredrickson[1], Mia Schneller[1], Mario Podestà[1], Raffi Nazikian[1], Mike Van Zeeland[4], David Pace[4] and Bill Heidbrink[5]

[1]Princeton Plasma Physics Laboratory, USA
[2]University of São Paulo, Brazil
[3]University of Texas, Austin, USA
[4]General Atomics, USA
[5]University of California, Irvine, USA

IAEA Energetic Particle Meeting – Princeton, September 5th 2017

Outline

- Introduction on Alfvénic spectral characteristics induced by energetic particles
- The cubic equation (Berk-Breizman model) and a criterion for onset of chirping
- Micro-turbulence as a mediator for chirping onset in DIII-D and NSTX
- Predictions for ITER

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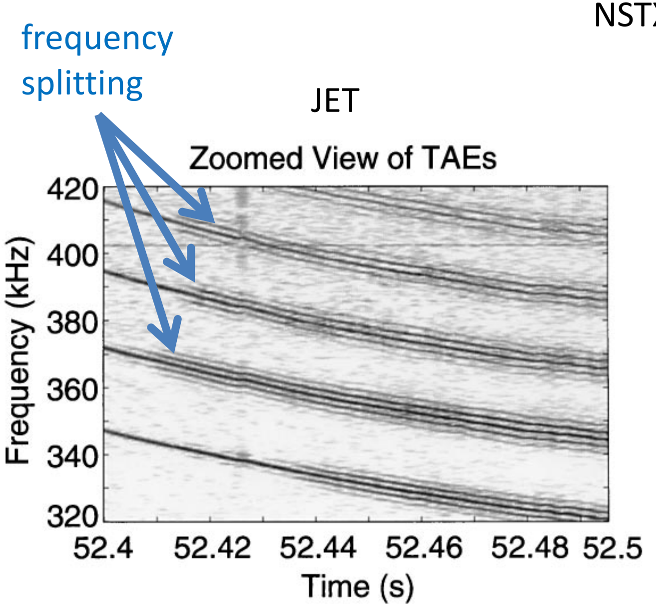
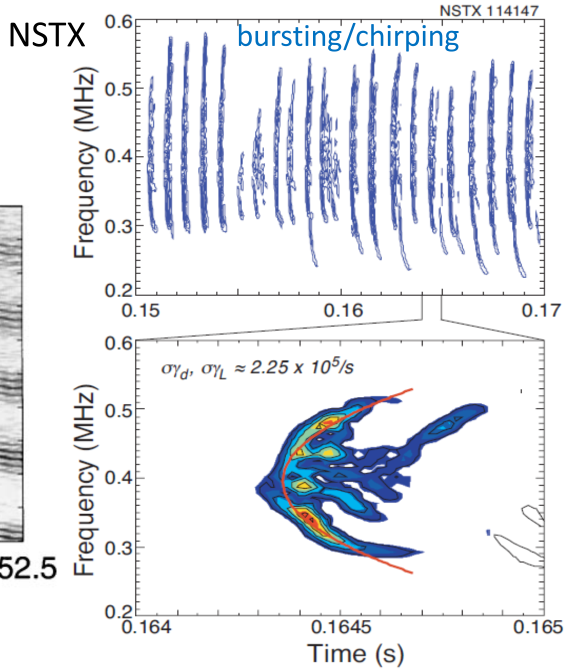
Alfvén waves can exhibit a range of bifurcations upon their interaction with fast ions

Typical scenarios:

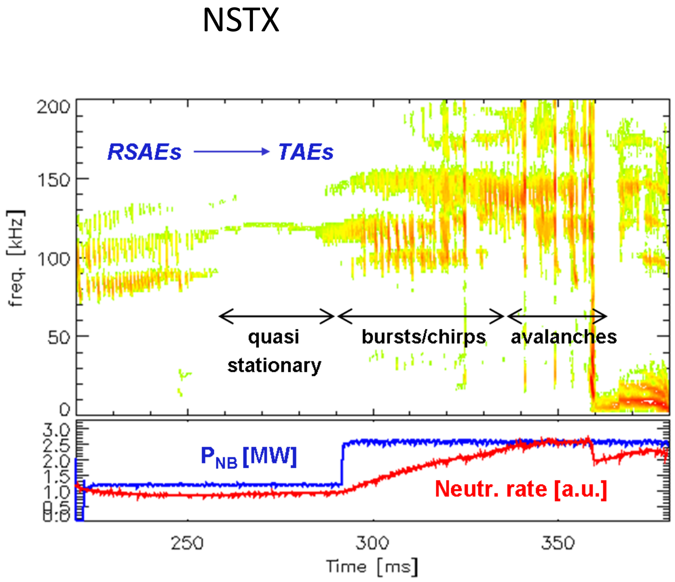
- fixed frequency and frequency splitting-> frequency is mostly determined by the equilibrium
- chirping and avalanches -> frequency is highly affected by the fast ions nonlinear response



Fasoli, PRL 1998



Fredrickson, PoP 2006



Podestà, NF 2011

Prediction of character of energetic-particle-driven transport in tokamaks

What tools can be used to model each type of transport?

Diffusive transport (typical for fixed-frequency modes)
- can be modelled using reduced theories, such as quasilinear
- typical in conventional tokamaks

Convective transport (typical for chirping frequency modes)
- needs to retain full nonlinear features of the wave, is sustained by nonlinear phase-space structures
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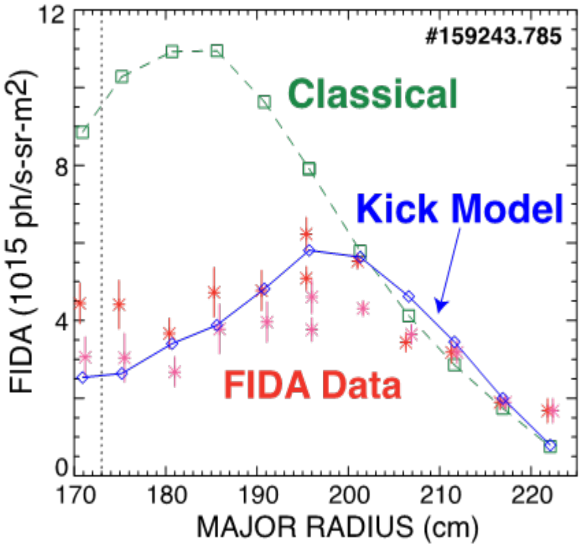
- **development of a criterion for the likelihood of each nonlinear scenario and its comparison with NSTX and DIII-D**
- **Predictions for TAE in ITER elmy and hybrid scenarios**

The study of the conditions that lead to fully nonlinear scenarios helps to understand the applicability of reduced models

Need for predictive/efficient interpretive capabilities motivates the development of phase-space resolved, self-consistent quasilinear approach



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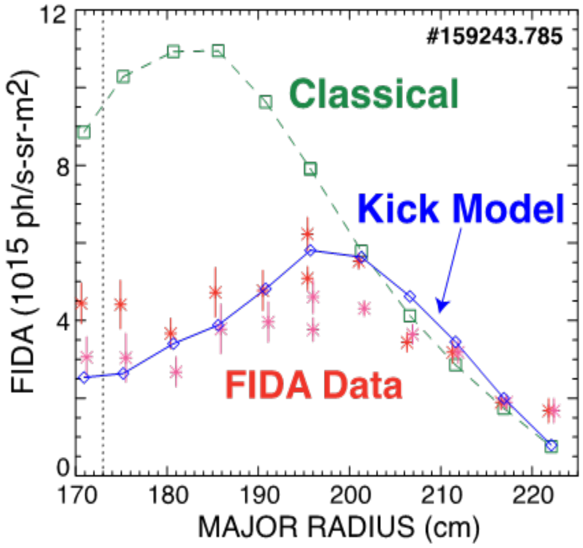
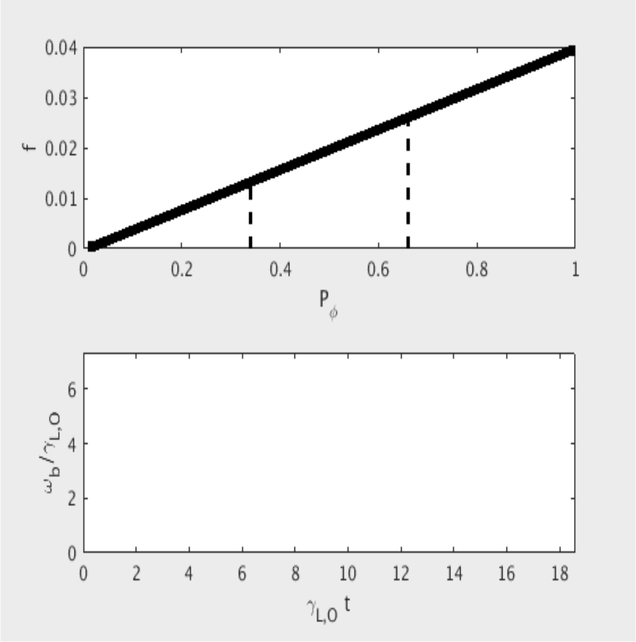


Diffusive module of the Resonance Broadening Quasilinear (RBQ) code, for the case of two overlapping resonances



[Detailed description of the RBQ code in N. Gorelenkov's poster]

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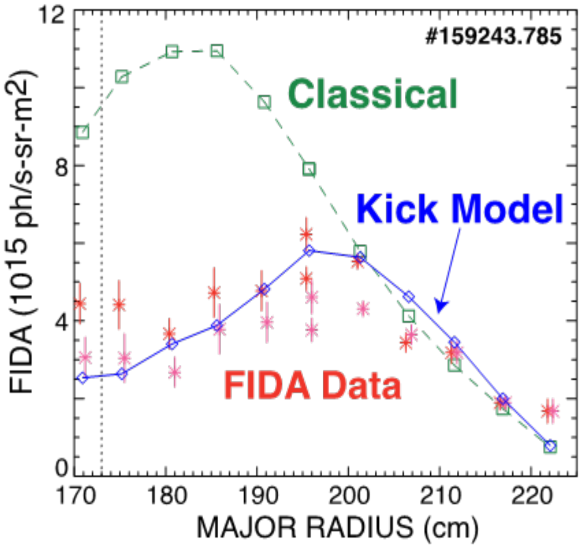

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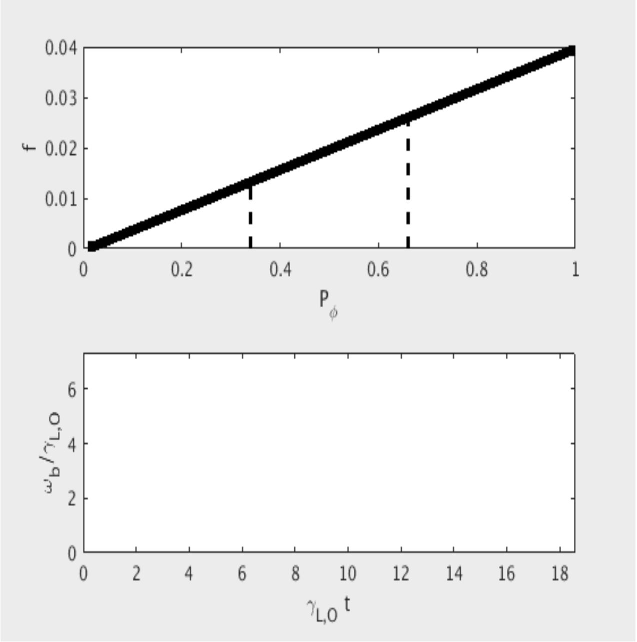

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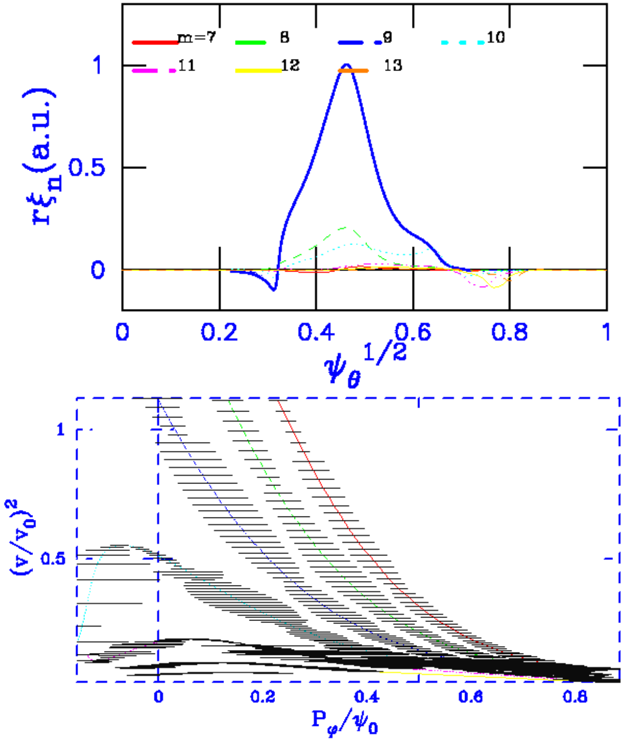
DIII-D discharge 153072



[Resonance broadening parametric dependencies in G. Meng's poster]

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Weak nonlinear dynamics of driven kinetic systems can be used to develop a criterion to distinguish between fixed-frequency and chirping responses

Starting point: **kinetic equation** plus **wave power balance**

Assumptions:

- Perturbative procedure for $\omega_b \ll \gamma$ ($\omega_b \propto \sqrt{mode\ amplitude}$)
- Truncation at third order due to closeness to marginal stability
- Bump-on-tail modal problem, uniform mode structure

Cubic equation: lowest-order nonlinear correction to the evolution of mode amplitude A:

$$\frac{dA}{dt} = A - \int_0^{t/2} d\tau \tau^2 A(t-\tau) \int_0^{t-2\tau} d\tau_1 e^{-\nu_{scatt}^3 \tau^2 (2\tau/3+\tau_1) + i\nu_{drag}^2 \tau(\tau+\tau_1)} A(t-\tau-\tau_1) A^*(t-2\tau-\tau_1)$$

Berk, Breizman and Pekker, PRL 1996 Lilley, Breizman and Sharapov, PRL 2009

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destabilizing (makes integral sign flip)

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- If nonlinearity is weak: linear stability, solution saturates at a low level and f merely flattens (system not allowed to further evolve nonlinearly).
- If solution of cubic equation explodes: system enters a strong nonlinear phase with large mode amplitude and can be driven unstable (precursor of chirping modes).

Berk, Breizman and Pekker, PRL 1996 Lilley, Breizman and Sharapov, PRL 2009

A criterion for the likelihood of chirping onset in tokamaks

Using an action and angle formulation, the previous weak nonlinear theory leads to

$$Crt = \frac{1}{N}\sum_{j,\sigma_\parallel}\int \mathrm{d}P_\varphi \int \mathrm{d}\mu \frac{\left|V_j\right|^4}{\omega_\theta \nu_{\mathrm{drag}}^4}\left|\frac{\partial\,\Omega_j}{\partial I}\right|\frac{\partial f}{\partial I} Int$$

$$Int \equiv Re\int_0^\infty dz \frac{z}{\frac{\nu_{stoch}^3}{\nu_{drag}^3}z - i} exp\left[-\frac{2}{3}\frac{\nu_{stoch}^3}{\nu_{drag}^3}z^3 + iz^2\right]$$

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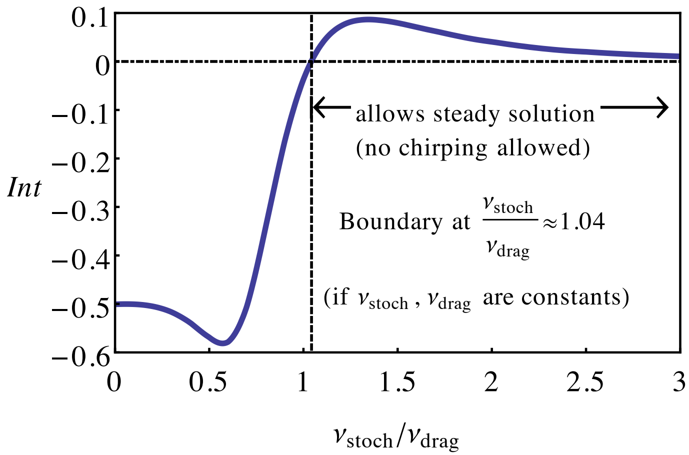

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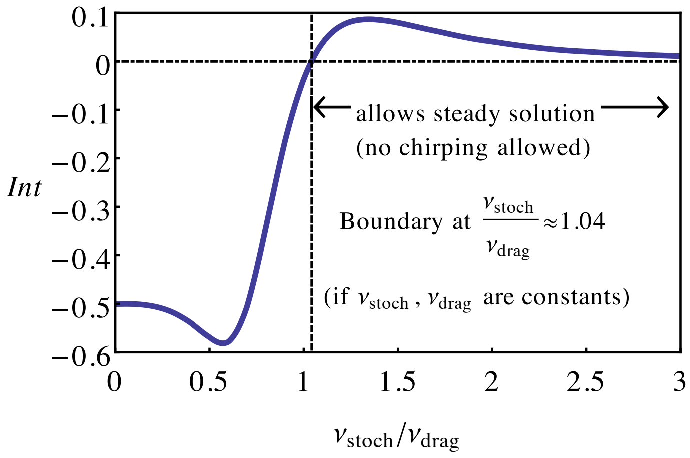

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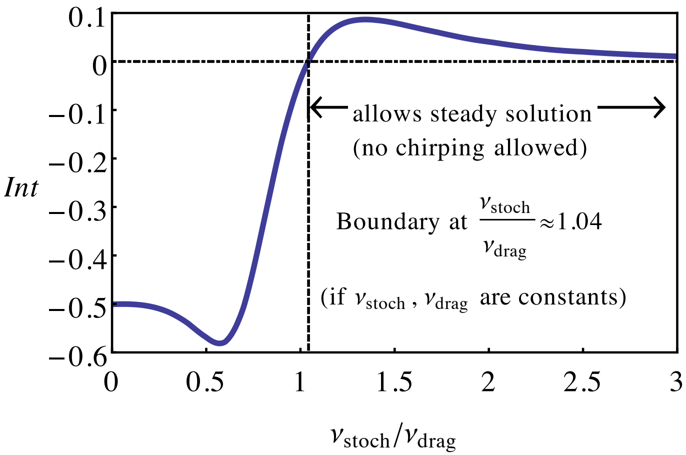
Eigenstructure information:

$$q\int dt\mathbf{v}_{dr}\cdot\delta\mathbf{E}e^{i\omega t}$$



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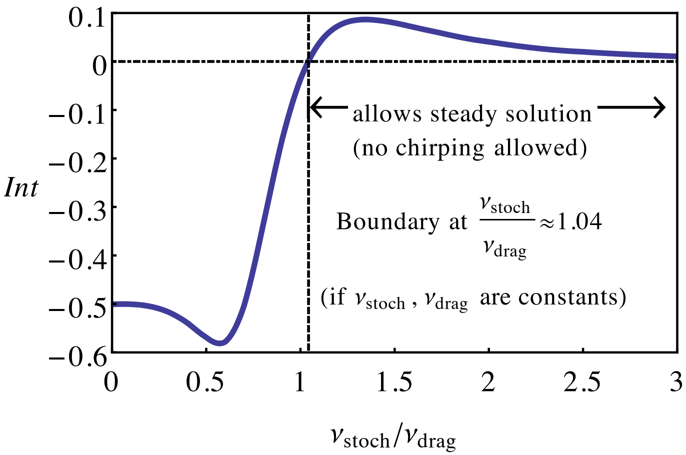

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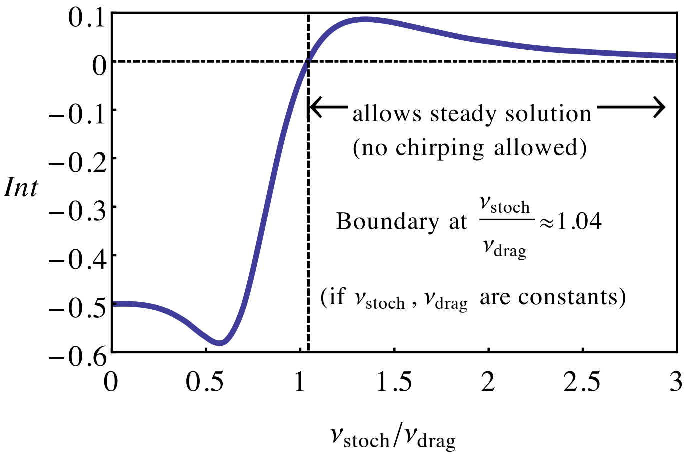

Criterion was incorporated into NOVA-K code: nonlinear prediction from linear physics elements

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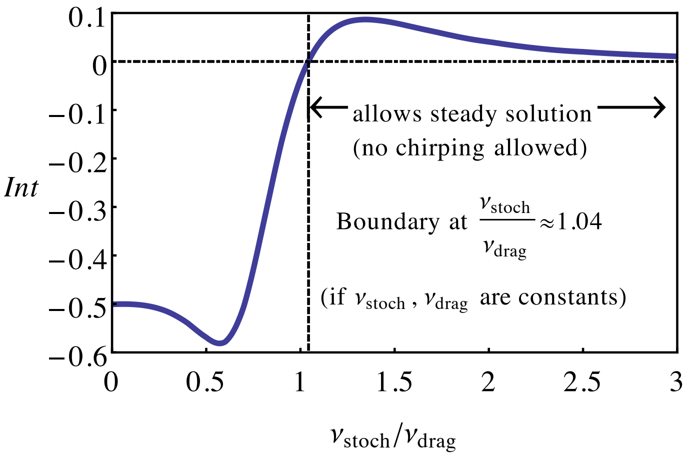

Criterion was incorporated into NOVA-K code: nonlinear prediction from linear physics elements

The criterion ($Crt \gtrless 0$) predicts that micro-turbulence should be key in determining the likely nonlinear character of a mode, e.g., fixed-frequency or chirping

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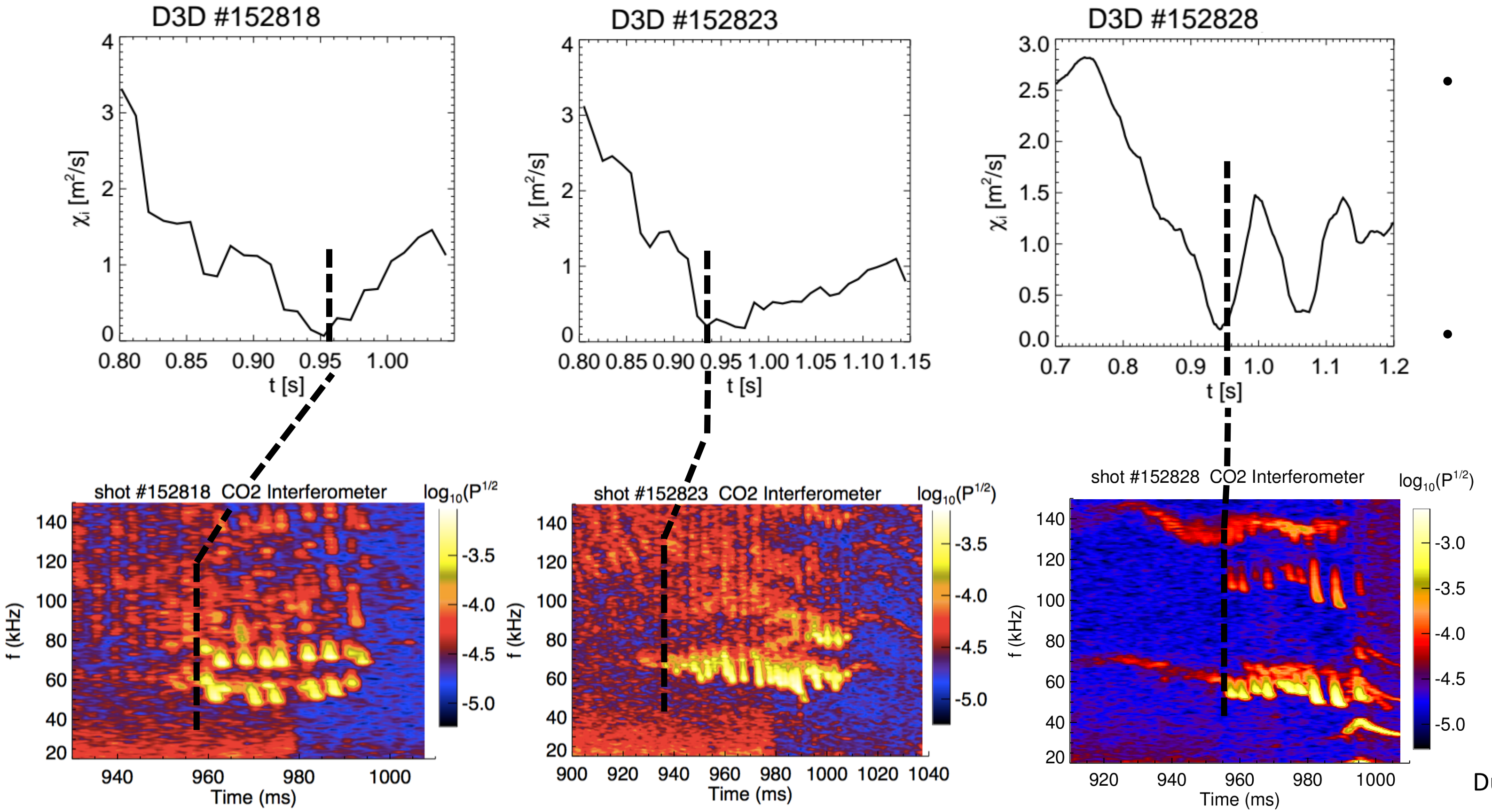
Correlation between chirping onset and a marked reduction of the turbulent activity in DIII-D



- The thermal ion heat conductivity is used as a proxy for the fast ion anomalous transport
- This observation motivated DIII-D experiments to be designed to further test the hypothesis of low turbulence associated with chirping

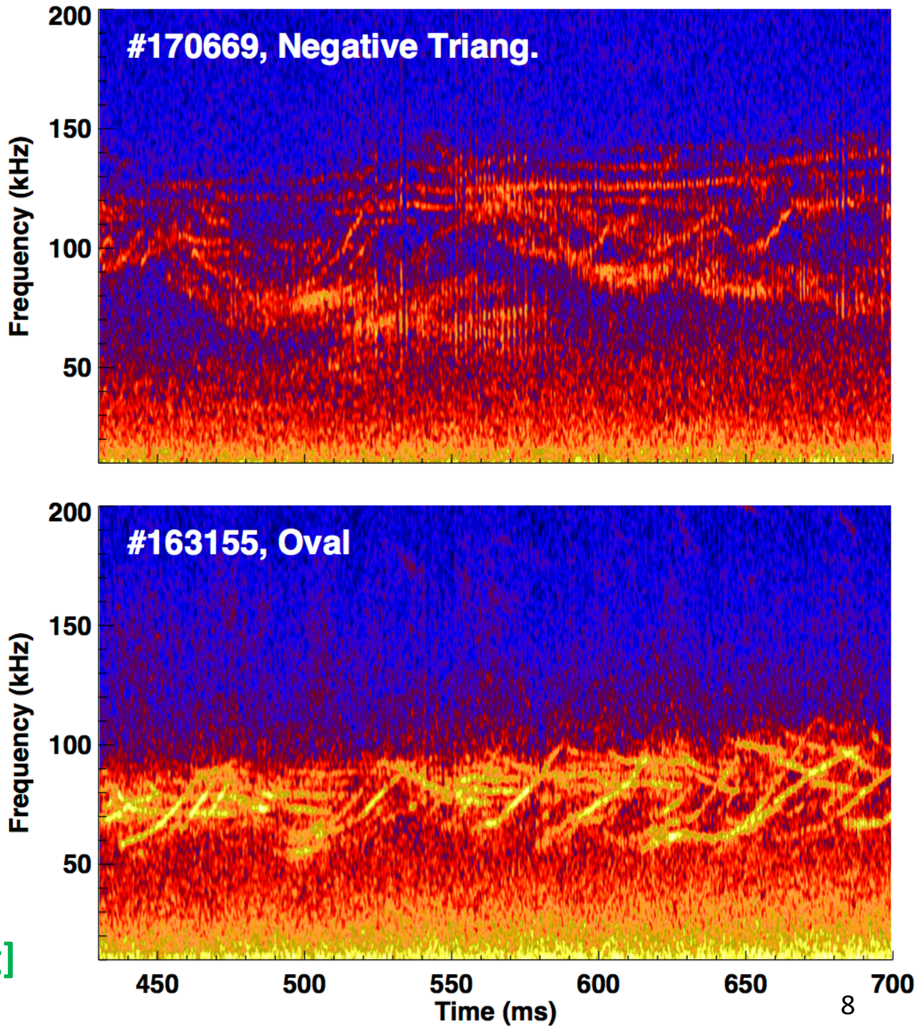
Duarte et al, NF 2017.

Dedicated experiments showed that chirping is more prevalent in negative triangularity DIII-D shots

- Transport coefficients calculated in TRANSP are 2-3 times lower in negative triangularity, as compared to the the usual positive triangularity/oval shots;

- Chirping found to exist in positive triangularity at the bottom of RSAEs evolution, where an ITB is expected.

[Detailed description of the negative triangularity experiments given in M. Van Zeeland's talk this morning]



DIII-D: chirping criterion evaluation in negative vs positive triangularity

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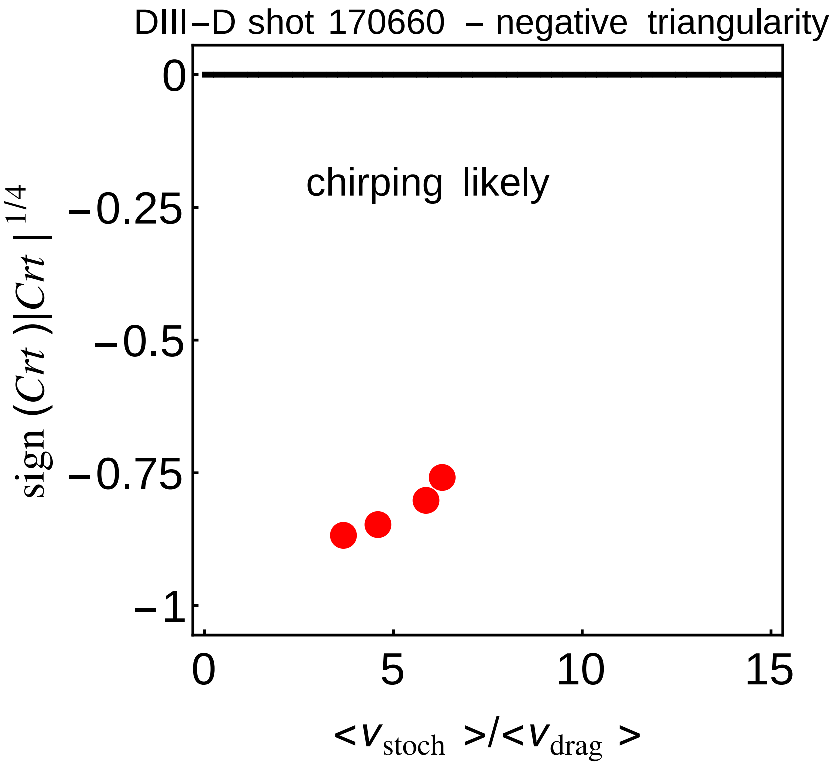

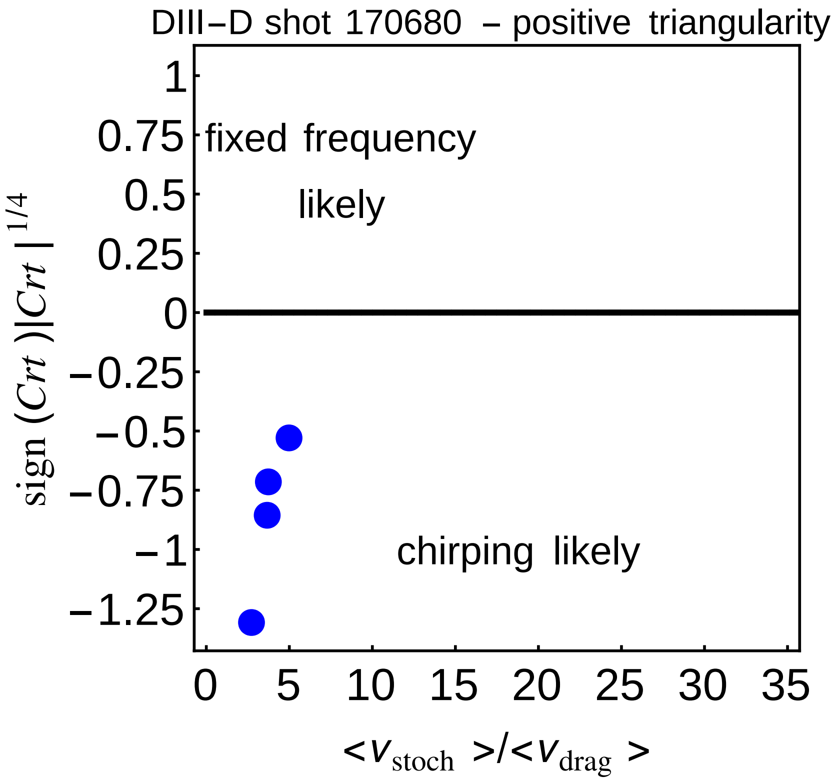

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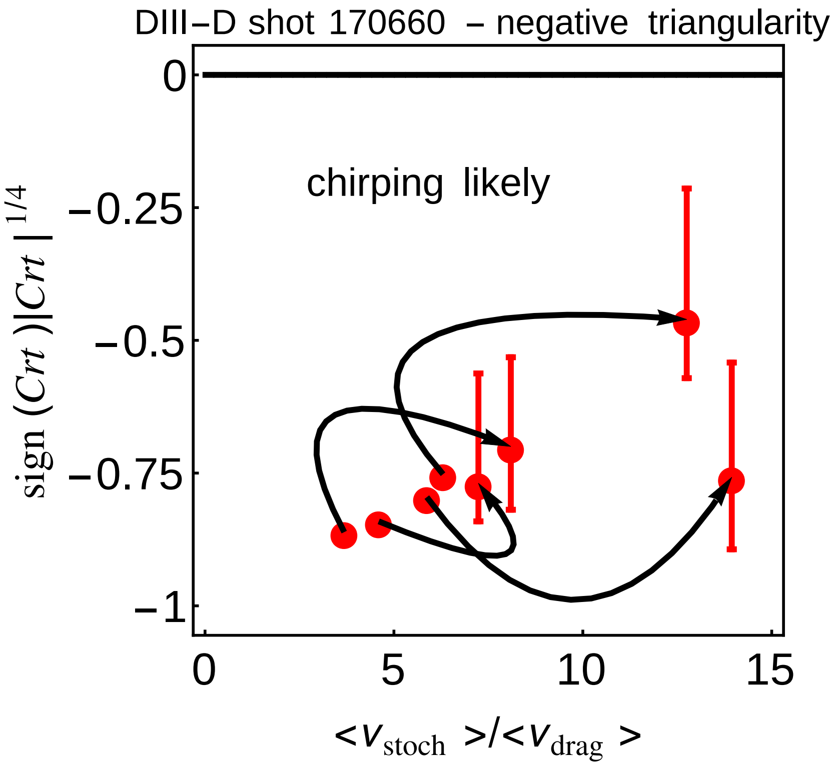
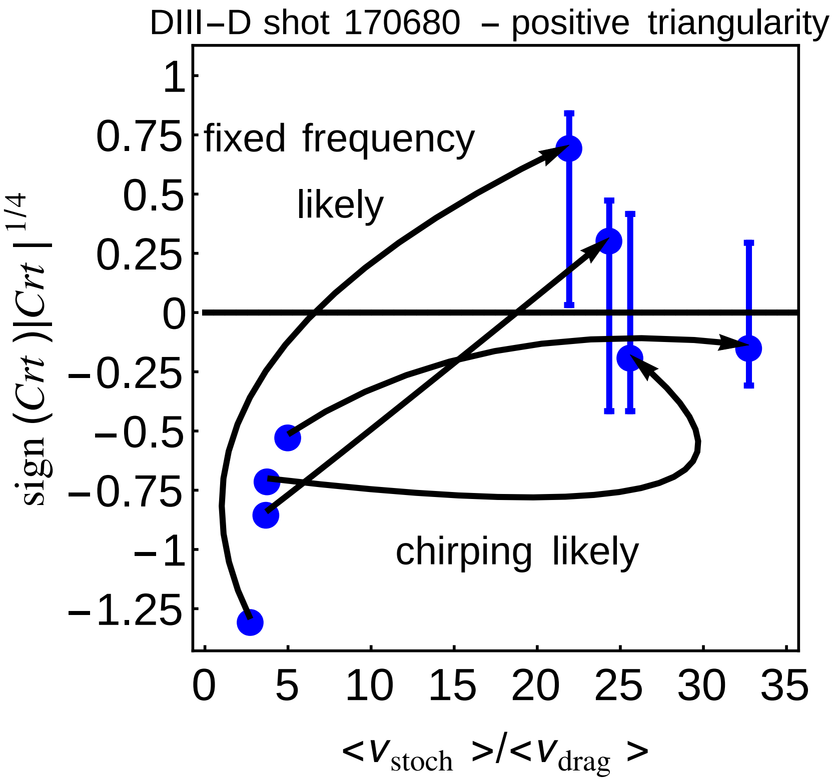
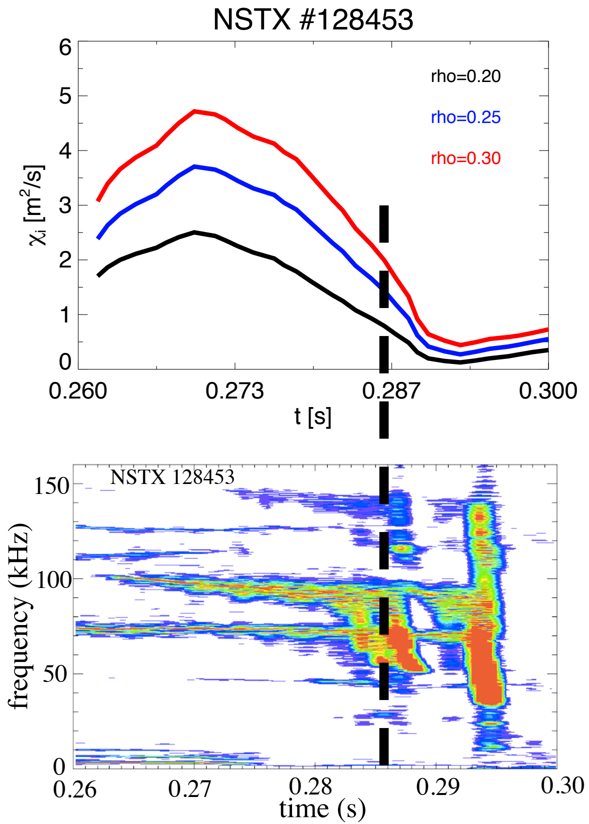
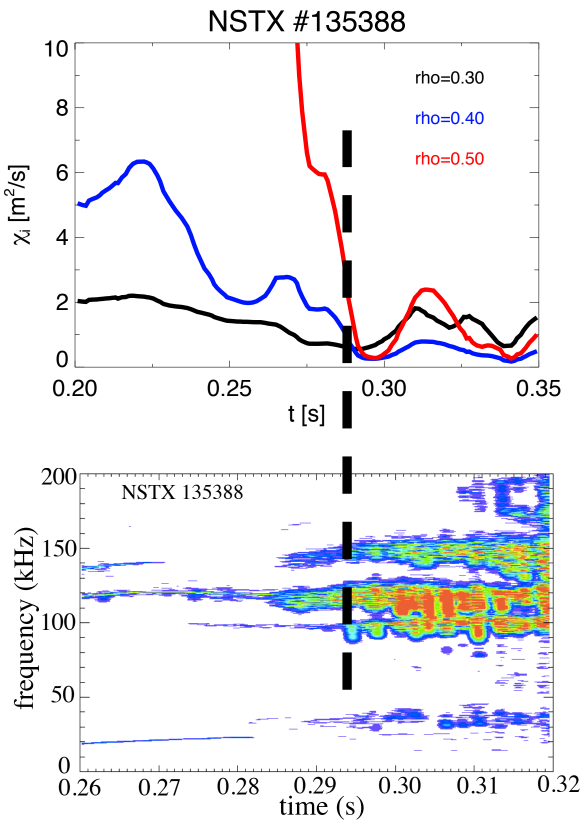




Correlation between chirping onset and a marked reduction of the turbulent activity in NSTX, as computed by TRANSP

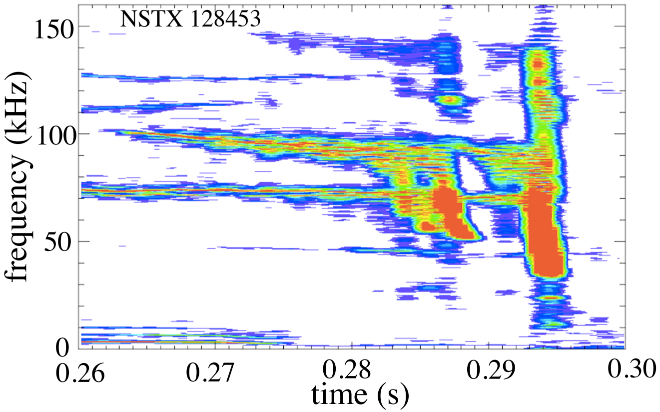


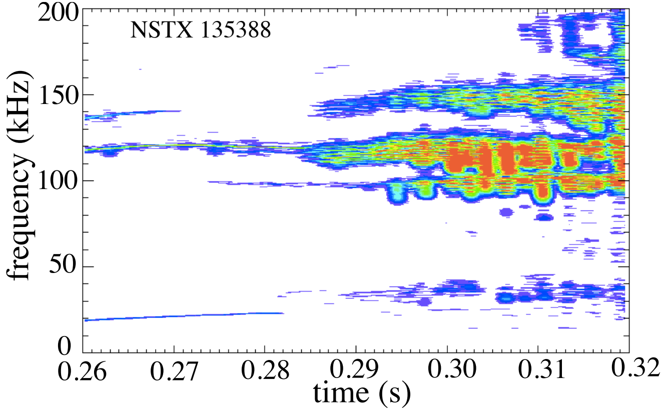


- The thermal ion heat conductivity is used as a proxy for the fast ion anomalous transport

- GTS code is being used to verify earlier publications on the EP turbulence-induced anomalous scattering

GTS* global gyrokinetics analyses show turbulence reduction for rare NSTX TAE transitions from fixed-frequency to chirping





*W.X. Wang et al., PoP **13**, 092505 (2006)

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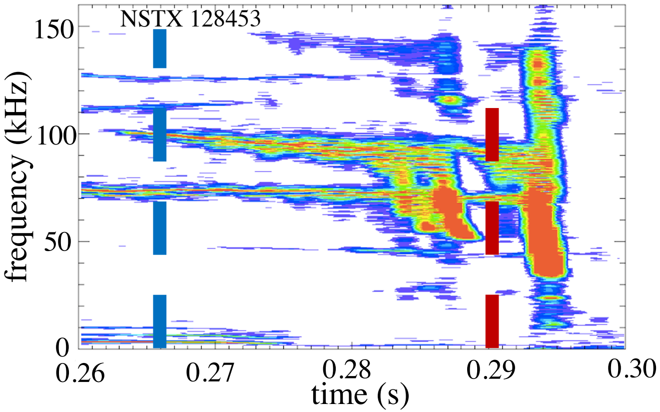

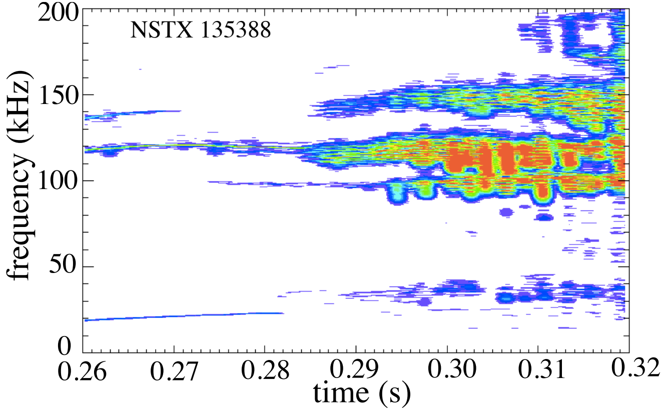

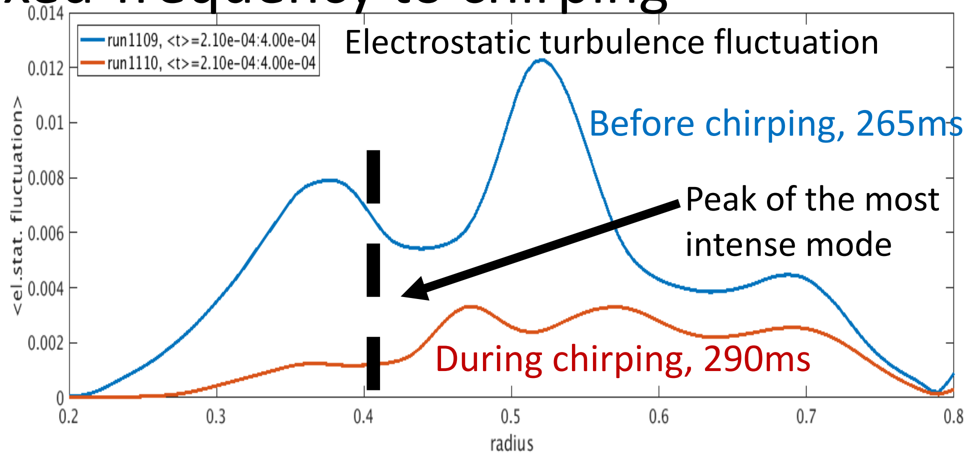

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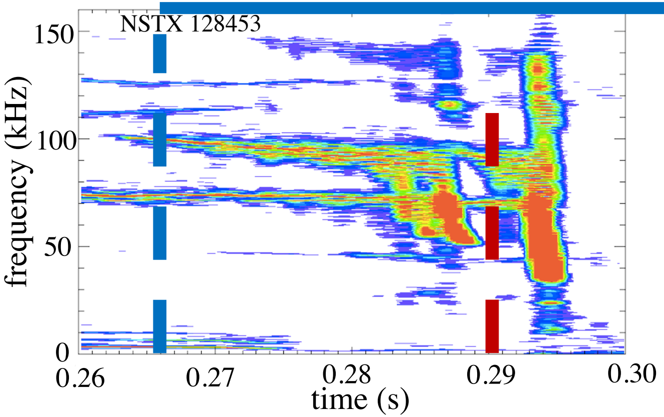
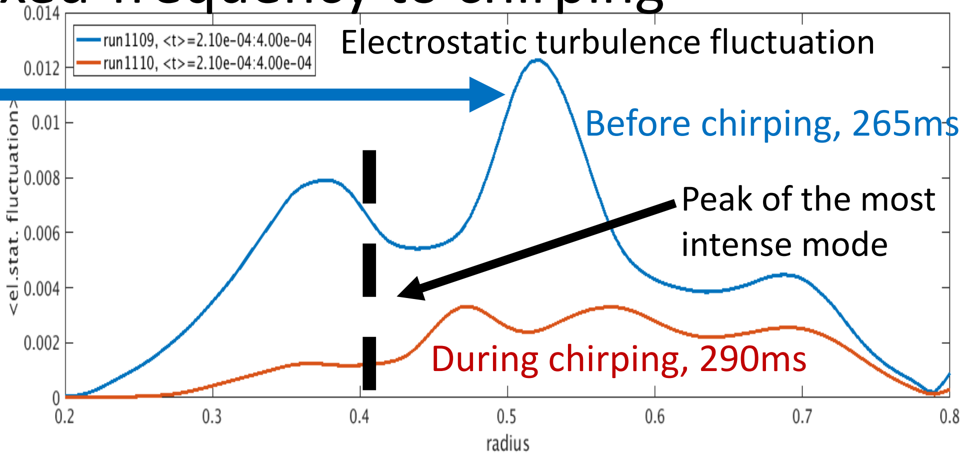
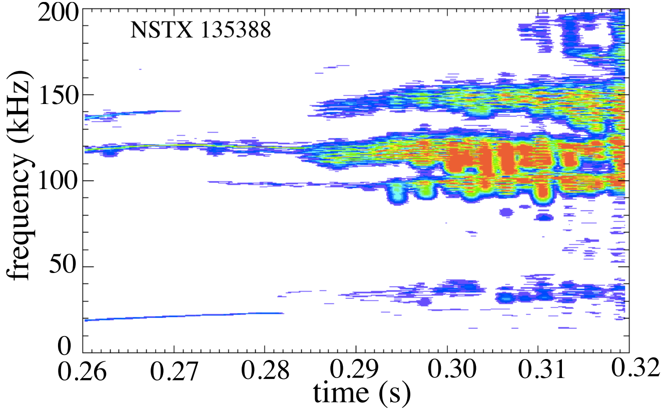
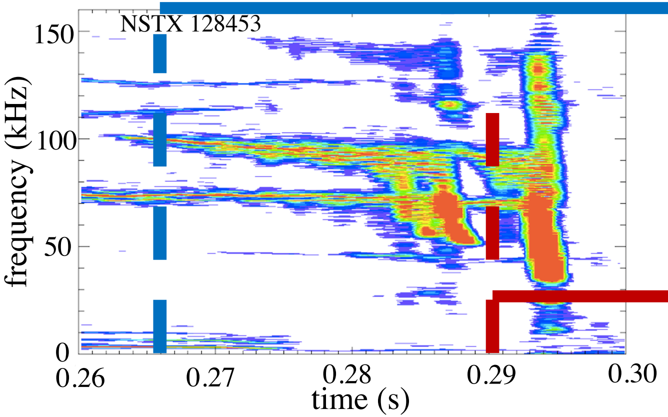


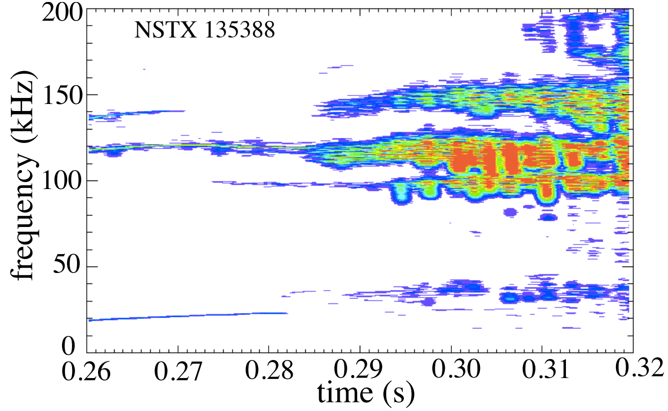


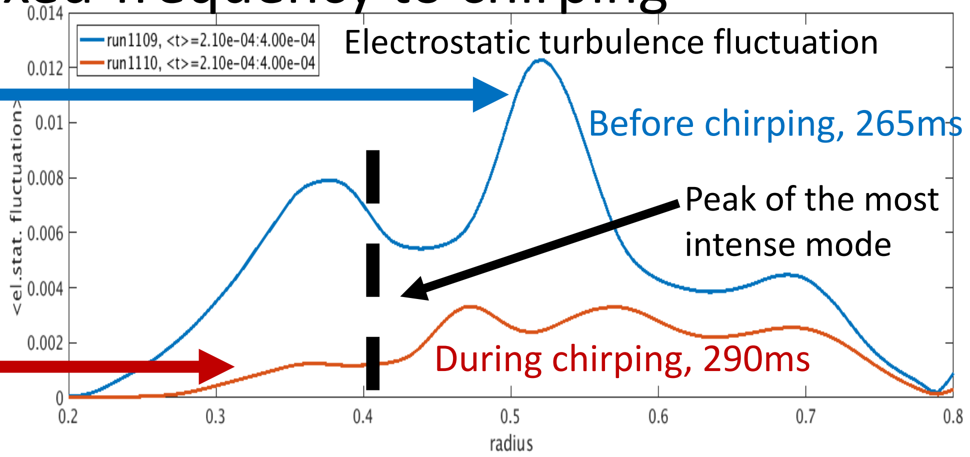


*W.X. Wang et al., PoP **13**, 092505 (2006)

GTS* global gyrokinetics analyses show turbulence reduction for rare NSTX TAE transitions from fixed-frequency to chirping







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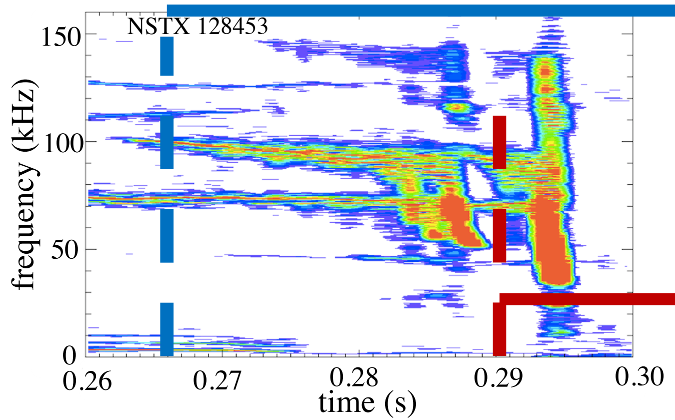

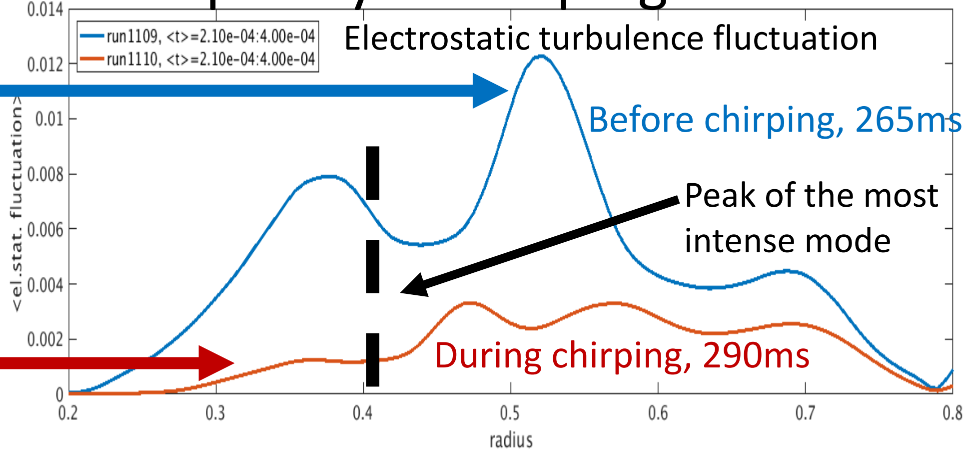

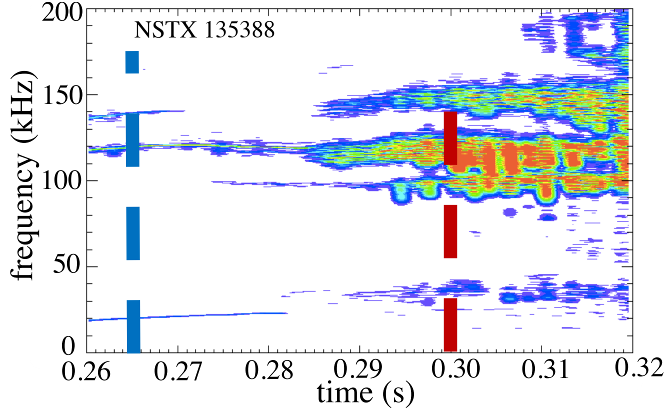

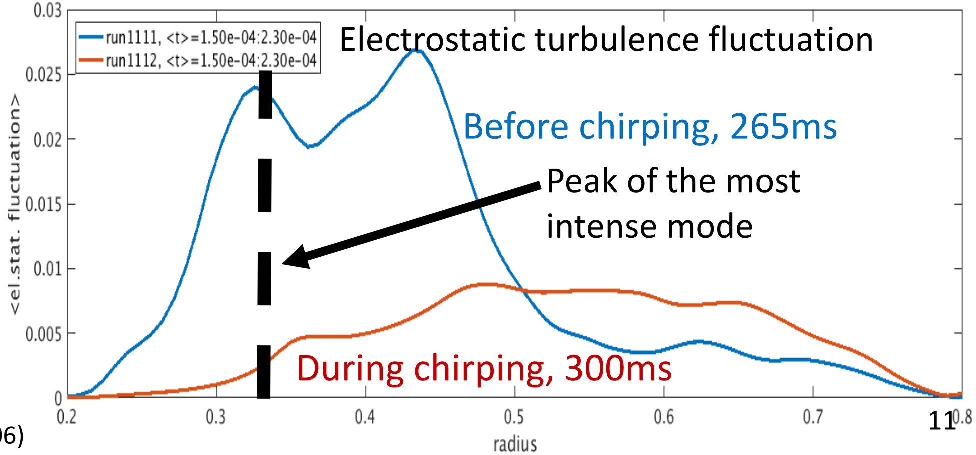

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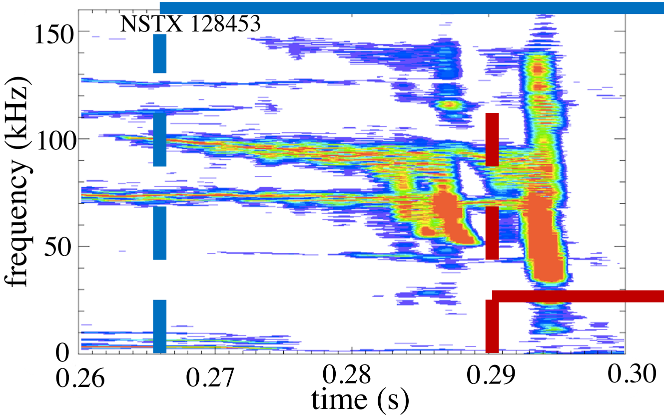

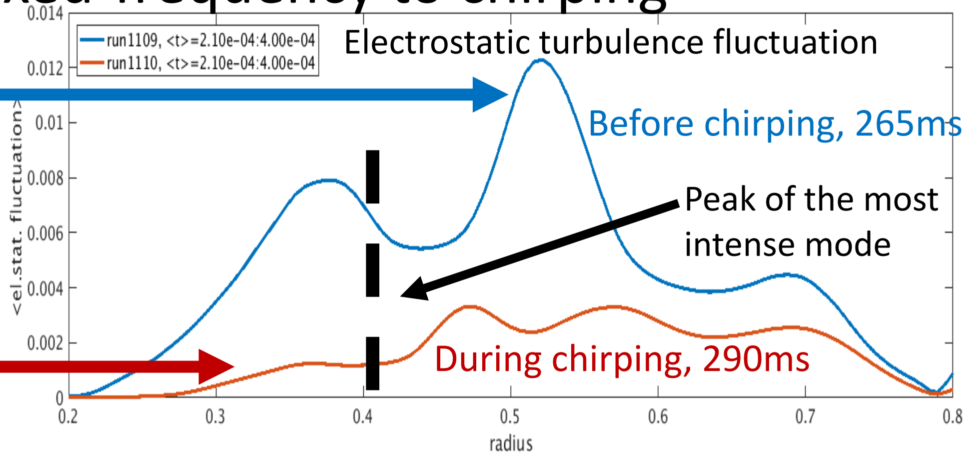

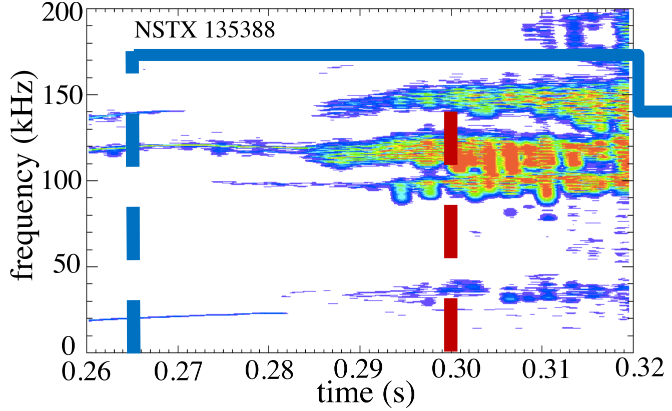

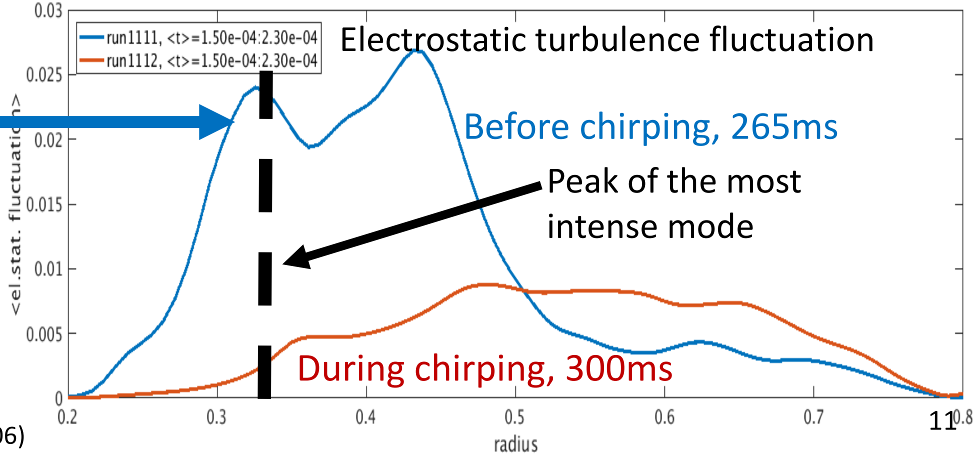

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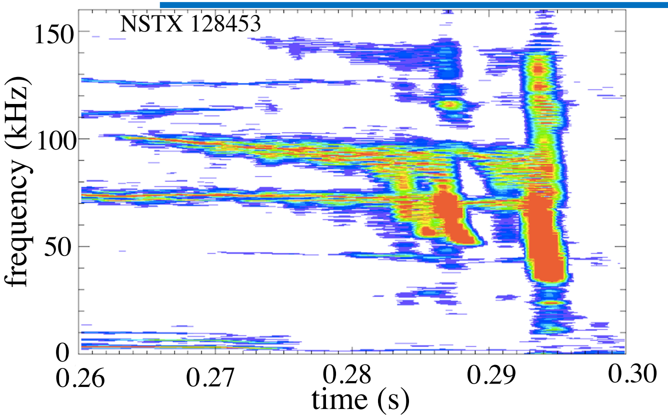

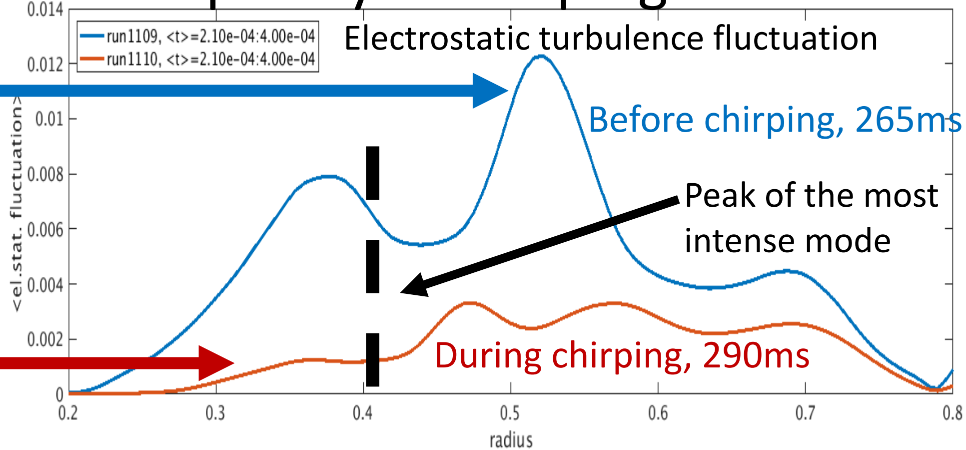

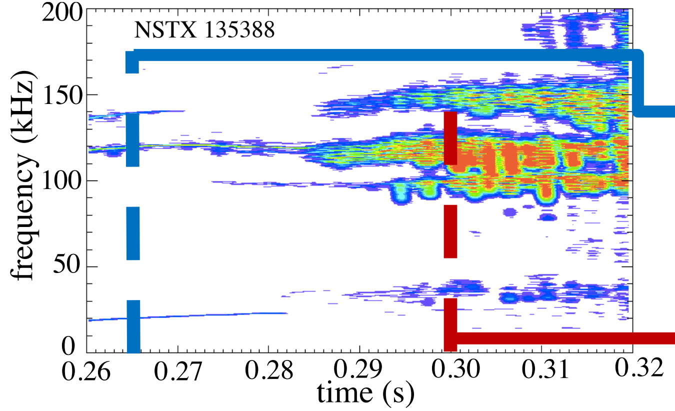

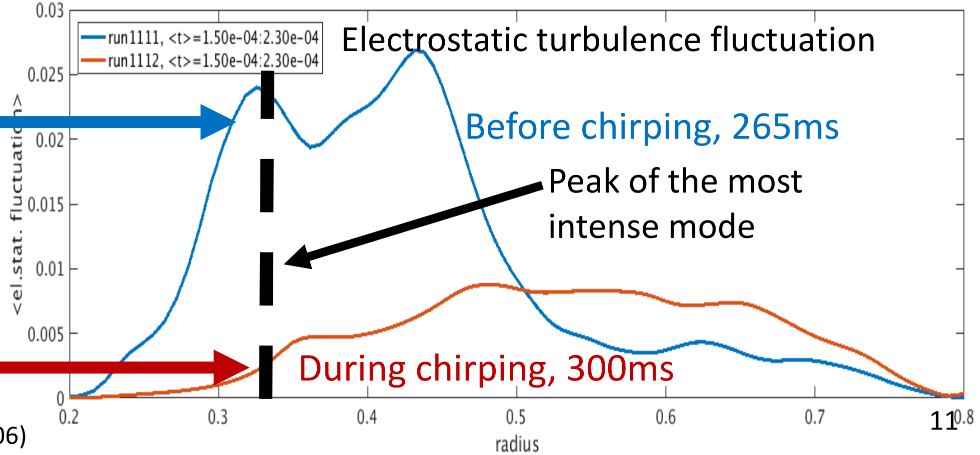

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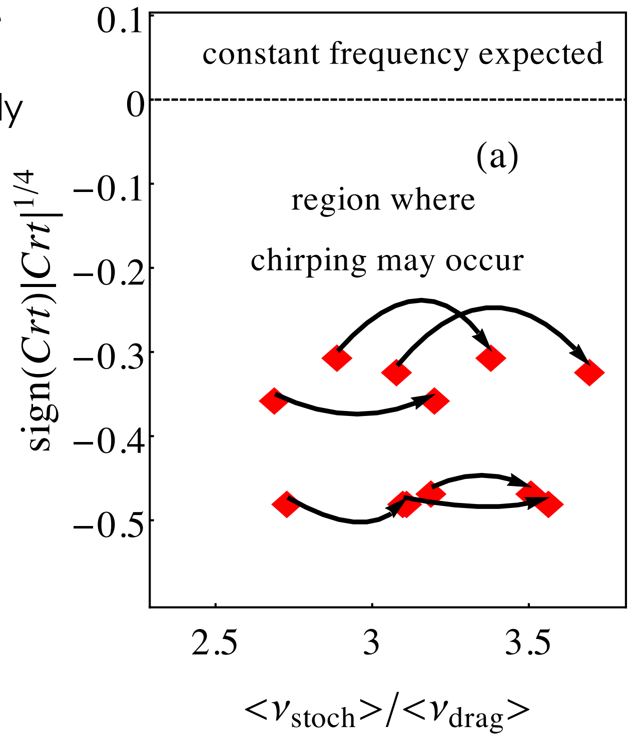
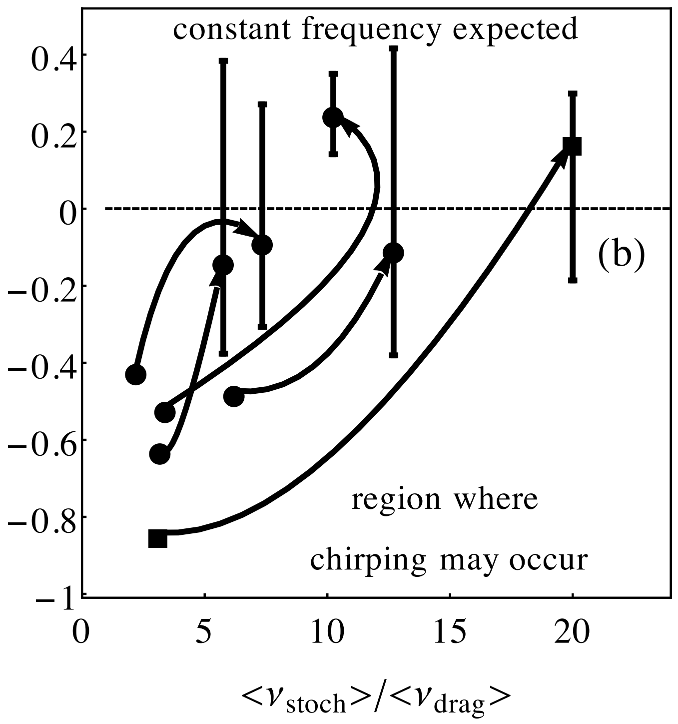
Examples of the chirping criterion evaluation: spherical vs conventional tokamaks

chirping, NSTX

fixed-frequencies, DIII-D and TFTR

Alfvén wave chirping quantitatively agrees with the criterion





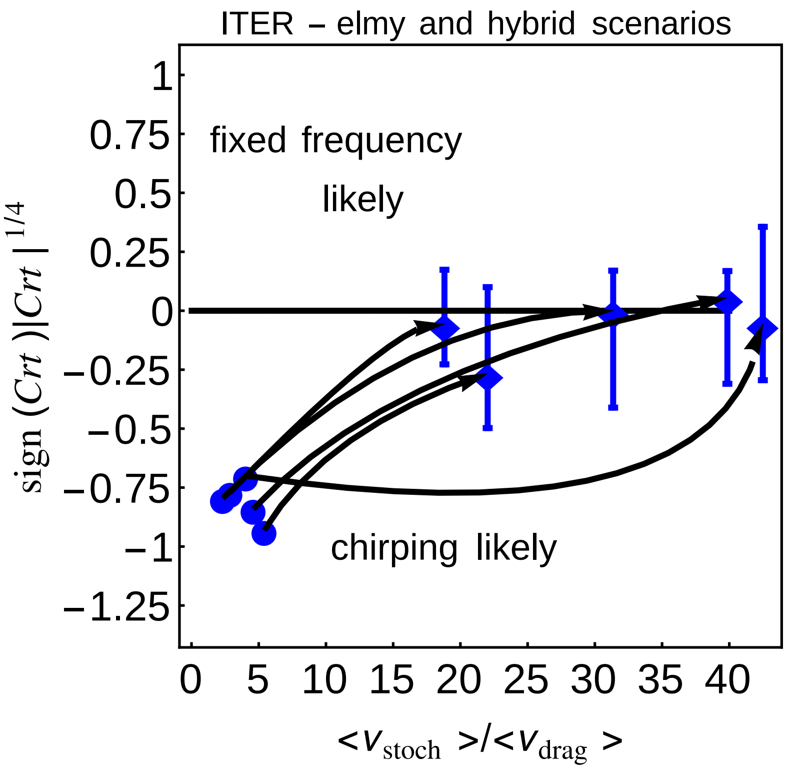
Arrows represent the turbulent diffusion that adds up to pitch-angle scattering

Chirping is ubiquitous in NSTX but rare in DIII-D, which is consistent with the inferred fast ion micro-turbulent levels

Outline

- Introduction on Alfvénic spectral characteristics induced by energetic particles
- The cubic equation (Berk-Breizman model) and a criterion for onset of chirping
- Micro-turbulence as a mediator for chirping onset in DIII-D and NSTX
- **Predictions for ITER**

Predictions for n=7-11 TAEs[1] in ITER are near threshold between steady and chirping



Based on TRANSP/TSC analysis, requiring Q>10

Approximate rate of radial turbulence diffusion to the collisional pitch-angle scattering[2,3]

$$Ratio \approx \frac{D_{EP}\left(\frac{q_{EP}}{m_{EP}}\frac{\partial\psi}{\partial r}\right)^2}{2\nu_\perp R^2\left[\mathcal{E}-\frac{B_\varphi^2}{B^2}\left(\mathcal{E}-\mu B\right)\right]}$$

$$\nu_\perp = \frac{1}{2}\langle Z\rangle\frac{\overline{A}_i}{[Z]}\frac{1}{A_{EP}}\left(\frac{v_c}{v}\right)^3\frac{1}{\tau_s}$$

[1] DOE OFES Theory Joule Milestone FY2007, Gorelenkov et al, PPPL Preprint number 4287 (2008).
[2]Lang & Fu, PoP 2010.
[3]Duarte et al, NF 2017.

Summary

- Criterion gives confidence in the application of quasilinear modeling;
- The gyrokinetic code GTS confirms transition from/to chirping is likely mediated by a change of turbulence;
- Experiments with negative triangularity on DIII-D give credence to the proposed chirping criterion predictions;
- Predicted response for ITER (similarly to DIII-D predictions) appears to be around the borderline between fixed-frequency and chirping.

Thank you